

## **Structural Reliability Studies on Reinforced Concrete Beam subjected to Shearing forces with Natural Stone as Coarse aggregate**

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### **Abstract**

This paper presents experimental and structural reliability studies performed on Bida natural stones (BNS) used as coarse aggregate in concrete production. Construction activities based on these naturally occurring materials are major steps towards industrialisation and economic integration of developing countries like Nigeria. This explains the huge interest over the years in utilising such materials as substitute or partial replacement for concrete constituents. The method adopted to appraise the aim and objectives of the study was laboratory experimentation. A total of 80 concrete cubes of 150 mm × 150mm × 150 mm were cast and used for this study. First order reliability method (FORM) was employed to ascertain the level of safety of the beam. Sensitivity analysis was further conducted by varying the span, effective depth and area of shear reinforcement of the beam in shear. The result revealed that the reinforced concrete beam utilising Bida natural stones is structurally safe at a span of 3000 mm and 3250 mm for unwashed and washed Bida natural stones respectively with probabilities of failure of  $1.07 \times 10^{-3}$  and  $1.14 \times 10^{-3}$ . Both unwashed and washed Bida natural stones were structurally safe at effective depth of 439 mm with probabilities of failure of  $5.87 \times 10^{-3}$  and  $1.31 \times 10^{-3}$  in shear.

### **Keywords**

Beam, Bida natural stones, sensitivity, shear, structural reliability

## 1. Introduction

Bida natural stone (BNS) is mostly found in Bida basin (Trough), it is a by-product of the Precambrian decomposition, transportation and deposition of rocks in this Basin. It is an extension of the Inllumedin Basin which runs through Niger Republic and Mali in West Africa. The Precambrian rocks in Northern Nigeria can generally be classified into four groups namely: Basement Complex, Older Granite series, Younger Metasediments, and Volcanic Rocks. This research investigates the effects of utilising washed and unwashed Bida natural stones in structural elements (Alhaji, 2016).

The need for naturally endowed content in the Nigerian construction industry is timely and justifiable. Bida natural stone is a naturally endowed occurring material abundantly available to the dwellers of the Bida basin. Construction activities based on these naturally occurring materials are major steps towards industrialisation and economic integration of developing countries. This explains the huge interest over the years in utilising such materials as substitute or partial replacement for concrete constituents (Aguwa and Sadiku, 2011).

Concrete is the most widely used construction material in the world, second to water as the most utilised substance on earth (Alhaji, 2016). It is obtained by mixing in the right quantities Cement, water, aggregates and sometimes with admixtures. Aggregates ideally constitute 75% of concrete hence are extremely important in determining the quality of concrete produced, this makes it important that they meet certain standards in order to achieve a strong, durable and economical concrete.

In any structure, there exist uncertainties or variability in loading and material properties. These uncertainties lead to variability in structural response during the life cycle of a structure (Jalayer *et al.*, 2011). In order to design structures that can perform the intended purpose with desired confidence, these uncertainties involved must be taken into account. The traditional way to tackle these uncertainties is to use the extreme values of the uncertain quantities and/or safety factors in the framework of deterministic design. However, a more accurate and precise way of treating the uncertainties is by utilising probability-based design methods that have been evolving and gaining widespread acceptability for the past few decades (Ayyub *et al.*, 1995; Wen, 2000).

Modern structural engineering advocates the application of structural reliability and probabilistic methods in analyzing structural materials, the goal of which is to produce a structure which is safe, economical, functional, elegant and meet client specification (Aguwa, 2013). Reliability is defined as "the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered". There are four basic elements to the definition that must be considered. First, probability refers to the likelihood that a structural component will work properly. These terms imply acceptance of some degree of uncertainty. The second element refers to adequate performance. In order to ascertain if a component has performed adequately, a standard is needed to specify what is meant by adequate performance. The third element is the intended period of time, this is the lifetime of the structure under consideration. The fourth element of the definition is the operating conditions. The environmental conditions play a large role in reliability of composite materials, particularly polymer matrix composites. Simply stated, structural reliability is a measure of the capability of a structure to operate without failure when put into service. In the broadest sense, structural reliability includes events that are safety and non-safety related (Chamis *et al.* 1993).

The use of Bida natural stones (BNS) for concrete production is gaining wide acceptance especially among the dwellers of the Bida basin because the production of crushed granite is labour intensive and expensive. BNS is used conveniently for mass concrete production and not commonly used for suspended reinforced concrete elements such as beams, columns or slabs because the structural reliability in that regard has not been evaluated. Salihu (2011) employed a constant water-cement ratio of 0.65 to produce concrete with BNS as coarse aggregate, the research determined just the compressive strength of the concrete at 3, 7, 14, 21 and 28 days of curing, the flexural strength, splitting tensile strength and elastic modulus were not considered. Furthermore, Alhaji (2016) developed models for predicting the compressive strength, flexural strength, splitting tensile strength and elastic modulus of concrete produced from BNS. Currently no research exists on structural reliability studies of concrete produced utilising this coarse aggregate. Based on this premise, reliability studies on BNS concrete becomes timely and justifiable.

The aim of this study is to conduct structural reliability studies on reinforced concrete beam subjected to shearing forces with natural stone as coarse

aggregate. The objectives are to; (i) determine the physical properties of fine and coarse aggregates. (ii) determine the 28 days compressive strength of cubes produced using unwashed and washed Bida natural stone and (iii) determine the reliability indices for the structural element.

## 2. Materials and Methods

Ordinary Portland Cement (OPC): The Cement used was obtained from the Building Materials Market Minna, Niger State. The bags of Cement were stored on a raised Platform where adequate protection from external effect was guaranteed. The OPC conforms to NIS 87:2004 and classified as CEM 1 from the standard.

Fine Aggregate : The sand was collected from Chanchaga river bed in Minna, Niger State. It was ensured that the sand was clean, sharp, free from clay and dirt's. Fine Aggregates generally refer to aggregates passing through sieve size 4.75 mm BS 882(1992).

Water: Water fit for drinking was sourced from the Civil Engineering Laboratory, Federal University of Technology Minna and used in casting the cubes. BS EN 1008 (2002) stipulates that water to be used for concrete production must be clean, drinkable and free from deleterious materials.

Coarse Aggregates:The Coarse Aggregates used for this work were sourced from Bida, Niger State, Nigeria. Bida is located in the Middle belt region of Nigeria. BNS lie in zone 31 and precisely fall within Latitude N 9° 55' E and Longitude N 5° 52' E. The coarse aggregate conforms to specifications for natural aggregates as in BS 882 (1992).

Casting of Concrete Cubes for Compressive Strength Test: The mix design employed in this study is the British Standard mix design (DOE) method. The Concrete specimens tested were cast in 150 mm x 150mm x 150 mm moulds for compressive strength test. The samples were thoroughly mixed with the aid of a Concrete mixer until the desired homogeneity of the mixture was achieved. Standard iron moulds of 150 mm x 150mm x 150 mm were used, it was ensured that the moulds were well lubricated with oil in order to reduce friction and enhance removal of the cubes from the mould. Each mould was then filled with concrete in three layers each tamped 25 times. 80 cubes were cast in total, 40 cubes for the unwashed BNS and 40 for the washed BNS. The cubes were cured for 28 days using Ponding method of curing. The cube casting was performed in accordance to BS 1881 (1983).

Compressive Strength Test: Compressive strength test on concrete cubes (80 Cubes) were determined using the ELE 2000 kN capacity Compressive

testing machine in the Civil Engineering laboratory of the Federal university of Technology Minna, Niger State. The weight of each cube was taken before crushing, this is however a destructive method of testing cubes. After crushing, the compressive strengths were calculated using Equation (1). The test was conducted in accordance with BS 1881:116.

$$F_{cu} = \frac{\text{Average Load}}{\text{Area}} \text{ (N/mm}^2\text{)} \quad (1)$$

## 2.1 Reliability Assessment

First-Order Reliability Method (FORM) was utilised in assessing the reliability of the structural element under consideration. This method is a simplified reliability model, it uses only the Mean values and Standard deviations for load and resistance variables in a particular Limit state in order to obtain the reliability index. It is a simplified model introduced first in structural steel design. The knowledge of type of distribution for the random variables is not needed for this analysis, hence all variables were assumed to be normally distributed. FORM is one of the most common basic techniques utilised for Structural reliability studies and is applicable to all probabilistic problems. It is usually preferred method, because it does not depend on the number of simulations to be carried out (Barambu *et al.*, 2017; Aguwa, 2013).

## 2.2 Sensitivity Analysis

Sensitivity analysis is the technique utilised in order to determine how independent variable values will impact a particular dependent variable under certain assumptions, it is also referred to as the what-if analysis. Sensitivity analysis was conducted by varying the span, effective depth and area of shear reinforcement separately without varying other variables so as to observe the effect of these variations on the beam.

## 2.3 Shear Failure

Table 1 shows a summary of input parameters utilised for the reliability analysis of the beam subjected to shear.

Table 1: Input variable for Shear Failure

Input Parameter	$X_i$	Mean	C.O.V	STD. DEV.	Distribution
Area of shear Reinforcement, $A_{sv}$ (mm <sup>2</sup> )	$X_1$	101	0.04	4.04	Normal
Breadth of beam, $b$ (mm)	$X_2$	225	0.05	11.25	Normal
Span of Beam $L$ , (mm)	$X_3$	4000	0.10	400	Normal
Dead load intensity, DL (kN/m)	$X_4$	17.70	0.20	3.54	Normal
Live Load intensity, LL (kN/m)	$X_5$	3.33	0.20	0.67	Normal
Spacing of shear reinforcement, $S_v$ (mm)	$X_6$	447	0.04	17.88	Normal
Effective depth of Beam, $d$ (mm)	$X_7$	359	0.10	35.9	Normal
Yield Strength of Steel, $f_y$ (N/mm <sup>2</sup> )	$X_8$	460	0.05	23.0	Normal
Comp. Strength $f_{cu}$ UW (N/mm <sup>2</sup> )		19.94	0.25	4.99	Normal
Comp. Strength $f_{cu}$ W (N/mm <sup>2</sup> )		22.52	0.25	5.63	Normal

C.O.V: Coefficient of Variation  
STD. DEV: Standard Deviation

UW: unwashed aggregate  
W: Washed aggregate

Shear force at the ends of a beam is given as:

$$V_u = \frac{1}{2} wL \tag{2}$$

The Shear resistance of the concrete beam is given by:

$$V_R = \left(\frac{A_{sv}}{S_v}\right) 0.87f_y d + b v_c d \tag{3}$$

Where  $A_{sv}$  = Area of shear Reinforcement,  $S_v$  = Spacing of shear reinforcement,  $f_y$  = Yield Strength of Steel,  $d$  = effective depth of beam,  $b$  breadth of beam and  $v_c$  = ultimate shear stress

**Performance function**

$$g_1(X) = V_R - V_u \tag{4}$$

Therefore the performance function for beam subjected to shearing forces is expressed in Equation (5)

$$g_1(X) = (0.87f_y d \frac{A_{sv}}{S_p} + b d v_c) - \frac{WL}{2} \tag{5}$$

**Performance function linearisation**

From Equation (5), substituting  $A_{sv} = X_1$ ,  $b = X_2$ ,  $L = X_3$ ,  $g_k = X_4$ ,  $q_k = X_5$  and linearising yields Equation 7.

$$g_1(X) = \frac{A_{sv}}{101} \times 0.87f_y 359 + b \times 0.55359 - \frac{1}{2} (1.4DL + 1.6 LL)L \tag{6}$$

$$= 1442.32X_1 + 197.45X_2 - 0.7X_3X_4 - 0.8X_3X_5$$

$$g_1(X) = 1422.32X_1 + 197.45X_2 - 15.054X_3 \tag{7}$$

Where DL = Dead Load and LL = Live load

**3. Results and Discussion**

Figure 1 presents the sieve analysis result obtained for fine aggregate. It can be seen that a Coefficient of uniformity ( $C_u$ ) of 3.5 was obtained. In order to classify a soil as well graded, the  $C_u > 6$  (Alhaji, 2016), hence it is concluded that the fine aggregate is not well graded.

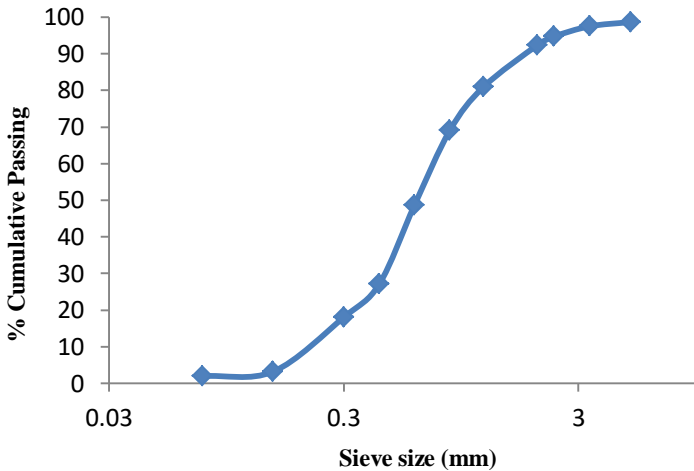


Figure 1: Sieve analysis result for Fine aggregate

### 3.1 Specific Gravity Test

Table 2 shows the results for specific gravity test performed on the Fine aggregate, an average specific gravity of 2.7 was obtained and is within the natural aggregates range of 2.6 – 2.7 (Neville and Brooks, 2008).

Table 2: Specific gravity result for Fine Aggregate

Trial	1	2	3
Weight of cylinder	115.0	116.5	116.6
Weight of cylinder + sample	207.2	240.6	248.9
Weight of cylinder + sample + water	487.3	512.0	484.9
Weight cylinder + water	428.0	435.7	402.2
Specific gravity of sample (S.G)	2.80	2.60	2.70
Average specific gravity (S.G)		2.70	

Table 3 presents the results for specific gravity test performed on the Coarse aggregate, an average specific gravity of 2.6 was obtained and is within the natural aggregates range of 2.6 – 2.7 (Neville and Brooks, 2008). This implies that the aggregate is suitable for construction work.

Table 3: Specific gravity result for Coarse Aggregate

Trial	1	2	3
Weight of cylinder	115.0	116.5	116.6
Weight of cylinder + sample	307.2	328.4	325.9
Weight of cylinder + sample + water	553.2	566.5	533.3
Weight cylinder + water	438.7	434.6	402.6
Specific gravity of sample (S.G)	2.50	2.65	2.66
Average specific gravity (S.G)		2.60	

### 3.2 Sensitivity Analysis

Figure 2 presents the result of sensitivity analysis conducted in shear for both unwashed and washed BNS varying the beam span (L). Increasing the beam span reduces the safety level with reliability indices lower than the 3.0 target reliability ( $\beta_T$ ) whereas decrease in the span increases the safety level for the concrete under consideration. The beam is reliable at span of 3000 mm for unwashed aggregate with reliability index of 3.07 and 3250 mm span for washed aggregate with reliability index of 3.05.



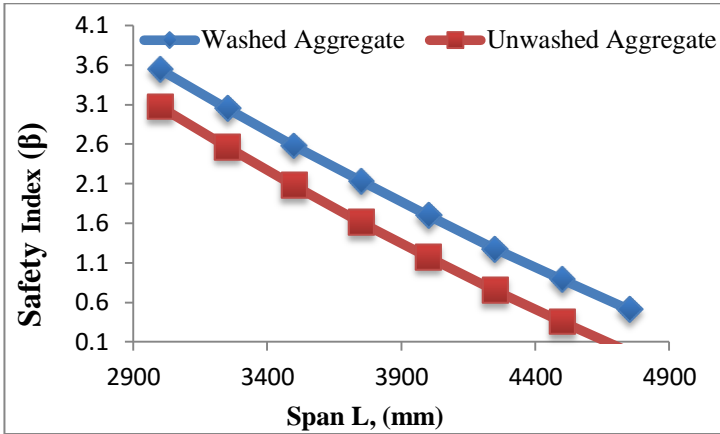


Figure 2: Relationship between Safety index and Span of beam in shear for washed and unwashed Aggregate

Figure 3 presents the result of sensitivity analysis conducted in shear for both unwashed and washed BNS varying the beam effective depth (d). A general increase in safety index (β) was observed as the depth was increased from. This increase in safety index (β) could be as a result of increased rigidity which translates into increased resistance in the beam.

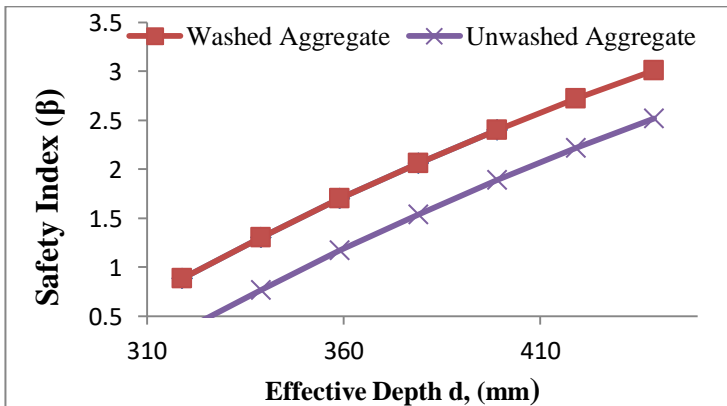


Figure 3: Relationship between Safety index and Effective depth in shear for washed and unwashed aggregate

As seen the washed aggregates returned higher reliability indices when compared with the unwashed aggregates.

Figure 4 presents the sensitivity analysis conducted in shear for both unwashed and washed Bida natural stones varying the Area of shear reinforcement ( $A_{sv}$ ). A general increase in safety index ( $\beta$ ) was obtained as the Area of shear reinforcement was increased. As seen from Figure 4 higher reliability index values were obtained for the washed aggregates when compared with the unwashed aggregates in shear, varying the  $A_{sv}$ .

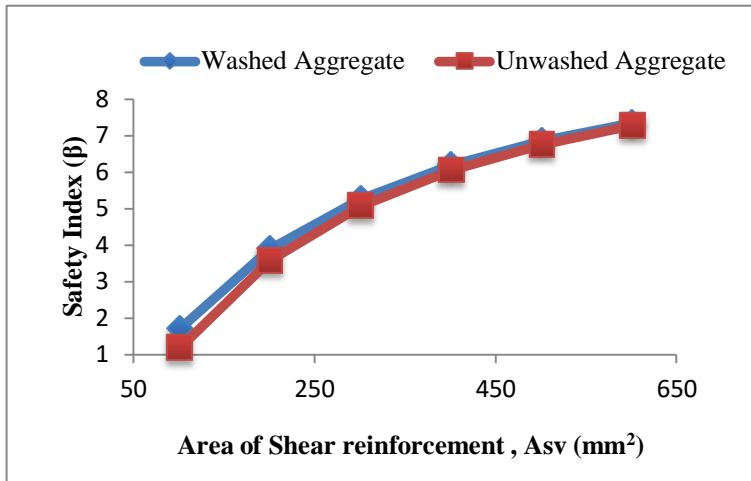


Figure 4: Relationship between Safety index and Area of Shear reinforcement ( $A_{sv}$ ) in shear for washed and unwashed aggregate

#### 4. Conclusion

The reinforced concrete beam with Bida natural stones as coarse aggregates is structurally safe against shear failure at a span of 3000 mm and 3250 mm for unwashed and washed aggregate respectively. The reliability index of the reinforced concrete beam with BNS subjected to shearing forces is highly sensitive to span and effective depth hence are the critical factors to be considered in the design of BNS beam. Generally, the structural performance can be improved by choosing shorter spans and larger depth.

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